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CIMAT

Centro de Investigación en Matemáticas, A.C.

1<sup>st</sup> and 12<sup>th</sup> July  
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MUFAMEX  
MAISON UNIVERSITAIRE FRANCO-MEXICAINE

EDUCACIÓN



MINISTÈRE  
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DE LA RECHERCHE  
ET DE L'INNOVATION

## ANNEX I

### Centro de Investigación en Matemáticas (CIMAT)

*Mechanisms that simplify stochastic nature*

#### 1. Framework

The study of stochastic phenomena emerging from the interaction of many sources of noise has gained enormous momentum in science in recent years. As a consequence, it has become vital to study mechanism that allow us to simplify the stochastic nature of a given problem. A fundamental aspect in the development of this topic is the study of theoretical metrics which could describe the precision error of the approximation of a random experiment composed of many small noisy signals with that of a simple probabilistic object. As a particular instance of this, we can mention the fundamental limit theorems in probability, associated to sums of independent and identically distributed random variables, which include the law of large numbers, the central limit theorem and the law of rare events. Nowadays, in many different areas of sciences, it is vital to extend such theorems to more general frameworks and contexts, such as the ones mentioned below. The problem to be addressed in this proposal consists on finding such approximating results motivated by concrete applications.

#### 2. Preliminaries

Although it is true that the relevance of the topic has growth substantially in the recent years, the study of it started developing long ago. In particular, we highlight the celebrated paper by Charles Stein 'A bound for the error in the normal approximation to the distribution of a sum of dependent random variables', which was published approximately 50 years ago. This paper laid the foundations of the so called "Stein's method", which refers to the collection of techniques that allow bounding probability distances by means of differential operators. This methodology has had a high impact in the study of limit theorems, which can be observed in the active scientific production related to it. The reader is invited to visit the URL <https://sites.google.com/site/steinsmethod/home> and <https://sites.google.com/site/malliavinstein> where we can consult an average of around 50 manuscripts per years related to subareas of Stein's method and limit theorems. Despite the admirable efforts that the scientific community has devoted to the study of limit theorems, there are still many open questions and subtopics to be studied, among which we can mention (i) Implementations of Stein's method in number theory (ii) The study of statistics emerging in the context of random graphs (iii) The study of limit distributions for statistics arising from Gaussian processes.

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### 3. Research statements

The project proposed in this document aims to contribute to the understanding of limit theorems in the spirit of the ideas presented by Charles Stein, focusing in the following particular subtopics 1 2 (i) The study of the relations between Stein's method and number theory: this topic has been only studied partially, and the results that had arose from the synergy between these two areas of mathematics have been quite promising. As evidence of this, the reader is referred to the manuscript 'A probabilistic approach to the Erdos-Kac theorem for additive functions', where a novel technique for studying problems in number theory by means of probability techniques, is presented. (ii) Stein's method and random graphs: recently, the study of statistics emerging in the context of random graphs has shown to be of great use in a variety of problems. Among them we can mention (1) Tests for determining the existence of patterns in models with interaction of many agents (2) description of topological properties of data by means of the study of certain statistics obtained from graphs associated to the date. As part of the research plan, it is proposed to address problems of similar nature, with a perspective of Stein method. (iii) The study of limit theorems for statistics arising in the context of Gaussian processes: regardless of the fact that this topic has been studied quite energetically during many years, the relevance of the topic for purposes of stochastic modeling, along with the enormous amount of open problems existing in the area, has turned this research area a crucial topic in science. The research proposal will put particular emphasis on the study of Gaussian functionals exhibiting dependence.

### 4. Proposal to the award team / Awardings

- To contribute to the theory of limit theorems by means of the elaboration of manuscripts devoted to the topic.
- To contribute to the appropriate formation of human resources by means of an adequate mentoring of students.
- To promote the consolidation of the Mexican mathematical community in the topics of limit theorems.

### 5. Authors of the issue

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